The application of the Full Markowitz Model in generating optimal investment portfolio

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Abstract: In this paper, the Full Markowitz Model will be used to compare different investment portfolios and make suggestions for investors on their investment decision-making process. By doing so, investors can have an overall view of the return and risks of their potential investments, and minimize risks or maximize return by choosing the optimal portfolio. To be more specific, in terms of minimizing risks, investing TD for 19.74%, PG for 28.52%, JNJ for 20.77%, and CL for 21% is the optimal portfolio. It has a return of 10.31%, a standard deviation of 11.25%, and a sharp ratio of 0.92. To maximize return, the weights of the optimal portfolio for Nividia Corporation, Cisco Systems, Intel Corporation, GS, U.S. Bancorp, TD Bank, The Allstate Corporation, P&G, J&J, and CL are 18.29%, 0.18%, -6.66%, 9.54%, 10.90%, 42.02%, 4.95%, 48.64%, 38.88%, and 18.25% respectively, with the return of 16.87%, standard deviation of 15.04%, and Sharpe ratio of 1.12.

1. Introduction

The Full Markowitz Model is applied very widely to provide optimal portfolios for investors. For instance, Manas Pandey [1] attempted "the optimal portfolio formation using real-life data subject to two different constraint sets" and "the Markowitz portfolio selections were obtained by solving the portfolio optimization problems for periods from April 2009 to March 2010." He further stated that "the final portfolio selection for an investor/trader requires the combination of portfolio analysis and financial planning [1]". Similarly, Mokta Rani Sarker [2] "construct an optimal portfolio by using Markowitz model" and "the optimum portfolio consists of twenty stocks selected out of 164 stocks, giving the return of 6.48%". However, Milica Radović, Snežana Radukić, and Vladimir Njegomir [3] "estimate the possibilities of the application of modern Markowitz's portfolio theory", and "identify the limiting factors for application on developing markets". For example, the limiting factors can be "illiquidity of the capital market in Serbia, which is the consequence of ownership concentration in the companies during privatization." Similarly, Myles E. Mangram [4] stated another limitation of the Full Markowitz Model, which is "the majority of investigations of the topic focus on the highly complex statistics-based mathematical modeling and formulas which support the concept's theoretical assumptions."

Although there are some limitations of the Full Markowitz Model, it is still helpful for investors to choose the optimal portfolio. As it is tested by Krishna Joshi and Dr. Chetna Parmar [5] on "whether the Markowitz Framework of Construction of Portfolio offers the improved Investment alternative to Investors of Indian Stock Market", it is concluded "from the Markowitz Selection of Portfolio criteria the best portfolio can be selected" under the Markowitz Framework. Also, Maria Laura Torrente and Pierpaolo Uberti [6] "analyzes the numerical stability of Markowitz portfolio optimization model", and found that "it is shown how standard portfolio optimization models can result in an unstable model also when the covariance matrix is well conditioned and the objective function is numerically stable" [6]. The usefulness of the Full Markowitz Model is further proved by Javier Vidal-García and Marta Vidal [7], "using a sample of daily mutual fund returns from the UK market for an interval covering from January 1st, 1990 to December 31, 2020", and found that "the Markowitz model can provide

portfolios that beat reference market portfolios, obtaining higher returns with a lower risk". Furthermore, some improved estimations to make Markowitz's portfolio optimization theory were introduced by Pui-lam Leung, Henry Ng, and Wing-Keung Wong [8] to solve the problem when return greatly overestimates the theoretical optimal return.

In this paper, firstly, background information of the stocks in the portfolio will be introduced and the trend of the stock price over the past five years will be illustrated. Then, the Full Markowitz Model will be explained and analyzed, including the relevant formulas and applications. After that, five different constraints will be discussed in detail and be compared according to their returns and sharp ratios to help to make conclusions. Finally, there will be a conclusion to summarize the research results of this paper and future improvements will be stated.

2. Data

The portfolio consists of ten stocks, which are Nividia Corporation, Cisco Systems, Intel Corporation, GS, U.S. Bancorp, TD Bank, The Allstate Corporation, P&G, J&J, and CL, during the time from May 2001 to May 2021. The performance of those ten stocks from May 2017 to May 2021 is shown in Figure 1.
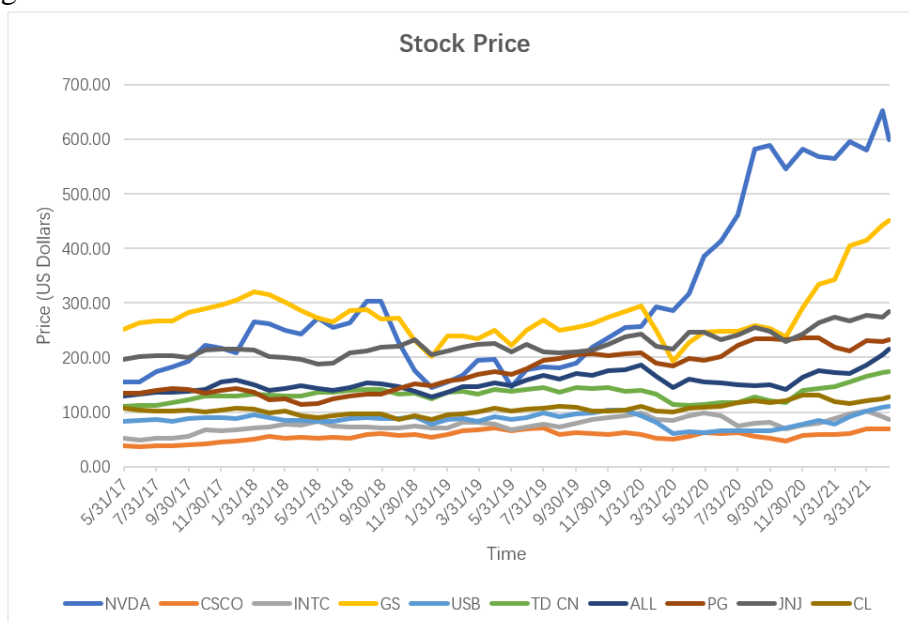

Figure 1 Stock Price

Nvidia Corporation is a US multinational technology company, which is the pioneer of GPU-accelerated computing. They specialize in products and platforms for the large, growing markets of gaming, professional visualization, data center, and automotive. As can be seen from figure 1, the stock price of NVDA increased significantly from 2020 to 2021, which was from $256.70 in January 2020 to $598.71 in May 2021.

Cisco is the worldwide leader in IT and networking. It manufactures and sells networking hardware, software, telecommunications equipment, and other high-technology services and products. It helps companies of all sizes transform how people connect, communicate, and collaborate. Compared to other stocks in this portfolio, CSCO has a relatively lower stock price which remained stable from 2017 to 2021.

Intel Corporation is a US multinational corporation and technology company that has a headquarter in California [9-10]. Intel is a world leader in the design and manufacturing of essential technologies that power the cloud and an increasingly smart, connected world. It is the world's largest semiconductor chip manufacturer. It is shown in figure 1 that the stock price of Intel fluctuated slightly during five years from $52.55 in May 2017 to $86.15 in May 2021.

The Goldman Sachs Group, Inc. is a leading global financial institution that delivers a broad range of financial services across investment banking, securities, investment management, and consumer banking to a large and diversified client base that includes corporations, financial institutions, governments, and individuals. The stock price of Goldman Sachs Group almost double from $246.04 in May 2020 to $450.59 in May 2021.

U.S. Bancorp is a US bank that is the parent company of the U.S. Bank National Association and is the fifth largest banking institution in the United States. The stock price of USB grew constantly from $62.92 in May 2020 to $109.68 in May 2021.

The TD Bank is a Canadian multinational banking and financial services corporation that has a headquarter in Toronto, Ontario. The bank was created in February 1955, through the merger of the Bank of Toronto and The Dominion Bank. The stock price of TD Bank increased gradually from $114.45 in May 2020 to $174.31 in May 2021.

The Allstate Corporation is a US insurance company. Founded in 1931 and the company also has personal lines insurance operations in Canada. The stock price of the Allstate Corporation showed an obvious increase in 2020, which is shown in figure 1.

P&G is a US multinational consumer goods corporation. It specializes in a wide range of personal health/consumer health, and personal care and hygiene products; these products are organized into several segments including Beauty; Grooming; Health Care; Fabric & Home Care; and Baby, Family Care. P&G's stock price grew gradually from $135.27 in May 2017 to $233.49 in May 2021.

J&J is a US multinational corporation founded in 1886 which develops medical devices and other consumer goods. It offers a large range of consumer healthcare products that include skin health, self-care, and essential health brands trusted by consumers and healthcare professionals worldwide. Here is one thing special, its common stock is a component of the Dow Jones Industrial Average and the company is ranked No. 36 on the 2021 Fortune 500 list of the largest US corporations by total revenue.

CL Company is a US multinational consumer products company headquartered in New York City. It specializes in the production, distribution, and provision of household, health care, personal care products. CL Corp's mission is to support companies along with their design process of interactive, creative, and innovative modules.

After introducing the ten stocks in the portfolio, betas of those ten stocks were calculated to illustrate the risk of the security. Beta measures the risk of the security relative to the risk of the market. For example, if there is a beta bigger than 1, it means the risk of the security has a relatively larger risk compared to the market. In this case, with a beta bigger than 1, the expected return of the security will be larger than just holding the market rate. To be more specific, beta also measures how the return of a security moves when the market return moves. It captures how sensitive the returns of an asset are to the market return. In this portfolio, NVDA has the largest beta, meaning that it holds almost double the risk than the market risk. For P&G, which has a beta of 0.41, it has a relatively lower risk than the market risk. In addition, alpha is the misplacing of stock. It is the difference between the actual return and the expected return. NVDA has the largest alpha which is 0.18.

3. Method

Using the Full Markowitz Model to exam portfolios under different constraints and compare the results to find out the optimal portfolio is useful for investors' decision-making process.

3.1 Full Markowitz Model

The Full Markowitz Model, related to Modern Portfolio Theory, is a portfolio optimization model, which was used to find out the most efficient portfolio that can maximize return or minimize risk for the investors. In the mean-variance framework, only two things matter for investors' decision-making process, which are expected return and the risk. In other words, in the mean-variance framework, when making decisions about a portfolio, investors only care about expected return and variance and they prefer higher returns and lower variance. By connecting the mean-variance framework with the investor's utility regarding risk and return, it is assumed that higher expected return increases utility

and higher standard deviation of returns decreases utility. The concept of utility helps to explain the intuition of the Full Markowitz Model.

The formula of MM portfolio return is, where w stands for a set of instruments' weight and u is a set of instruments' average return. The formula of MM portfolio standard deviation is P is the matrix of instruments' cross-correlation coefficients and v is an auxiliary vector.

In addition, to setup, the MM model, the annual average return, annual standard deviation, beta, alpha, residual standard deviation, and the correlation matrix of the ten stock and SPX were calculated.

3.2 Constraint

The solver in Excel can be used to minimize the variance or maximize the sharp ratio of the MM model. Firstly, the solver was asked to minimize the standard deviation of the MM model by changing the weights of the stocks. There are five different constraints. The first constraint is to let the sum of the weight of the ten stocks be less than two, the second is to set the absolute value of the weight of the stock to be less than one, the third situation is no constraints, the fourth one is to let the weight of stock not to be negative, and the last constraint is to let w1 equal to zero. The same procedure under five different constraints was also conducted to maximize the sharp ratio.

4. Result analysis

The minimum variance portfolio (MVP) defines the lowest possible risk and therefore the minimum expected return that the investor is willing to accept. The shape ratio of a portfolio is obtained by dividing the risk premium by the standard deviation. It is a measure of risk premium per unit of standard deviation and investors prefer assets with higher Sharpe ratios.

To be more specific, weights for each stock, return, standard deviation, and Sharpe ratio of the portfolio under five different constraints are shown in Table 1. Firstly, under the first constraint, which is making weights of the ten stock to be less than two. When setting the goal to minimize the variance, the weights for Nividia Corporation, Cisco Systems, Intel Corporation, GS, U.S. Bancorp, TD Bank, The Allstate Corporation, P&G, and J&J, and CL are -3.04%, -3.00%, 1.32%, -6.08%, -0.44%, 19.33%, -11.33%, 25.51%, 18.85%, and 19.04% respectively. The portfolio under the first constraint has an average return of 8.91%, a standard deviation of 10.93%, and a sharp ratio of 0.82. When setting the goal to maximize Sharpe ratio, the weights for those ten stocks are 14.28%, -1.09%, -5.44%, 3.66%, 6.72%, 35.15%, 0.79%, 43.96%, 30.98%, and 14.45%, with return of 15.06, standard deviation of 13.59%, and Sharpe ratio of 1.11.

Secondly, when the constraint is setting the absolute value of the weight of the stock to be less than one and the goal of the solver is to minimize variance, the weights of the portfolio are -3.04%, -3.00%, 1.32%, -6.08%, -0.44%, 19.33%, -11.33%, 25.51%, 18.85%, and 19.04% respectively, which has an average return of 8.91%, standard deviation of 10.93%, and a sharp ratio of 0.82. If the goal is to maximize Sharpe ratio, the ten stocks are allocated under weights of 18.29%, 0.18%, -6.66%, 9.54%, 10.90%, 42.02%, 4.95%, 48.64%, 38.88%, and 18.25%, with return of 16.87%, standard deviation of 15.04%, and Sharpe ratio of 1.12.

Thirdly, when there is no constraint, the weights of the ten stocks are -3.04%, -3.00%, 1.32%, -6.08%, -0.44%, 19.33%, -11.33%, 25.51%, 18.85%, and 19.40% respectively, which has an average return of 8.91%, standard deviation of 10.93%, and sharp ratio of 0.82. If the goal is to maximize sharpe ratio, the portfolio has weights of 18.29%, 0.18%, -6.66%, 9.54%, 10.90%, 42.02%, 4.95%, 48.64%, 38.88%, and 18.25%, with return of 16.87%, standard deviation of 15.04%, and sharpe ratio of 1.12.

Fourthly, under constraint four which is letting the weight of stock not to be negative, when the goal is to minimize variance, the weights for those ten stocks are 0.00%, 0.00%, 0.00%, 0.00%, 0.00%, 19.74%, 0.00%, 28.52%, 20.77%, and 21.00%, with return of 10.31%, standard deviation of 11.25%, and sharpe ratio of 0.92. If the goal is to maximize sharpe ratio, the portfolio has weights of 9.32%, 0.00% , 0.00%, 0.00%, 0.00%, 23.96%, 0.00%, 40.43%, 17.53%, and 8.76% with return of 13.06%, standard deviation of 12.66%, and sharpe ratio of 1.03.

Fifthly, when the constraint is making w1 equal to zero and the goal is to minimize variance, the weights of those ten stock are -0.98% , 0.07%, 2.54%, -1.04%, 3.47%, 24.76%, -7.90%, 28.58%, 25.78%, and 24.73% respectively, with the return of 10.15%, standard deviation of 11.17%, and sharpe ratio of 0.91. When the goal is to maximize sharpe ratio, the weights are 12.64%, -5.88%, -8.30%, -1.30%, 2.55%, 29.86%, -2.85%, 40.91%, 24.10%, and 8.27%, with return of 13.88%, standard deviation of 13.06%, and sharpe ratio of 1.06.

To find the optimal portfolio for the investor, we need to find the portfolio that maximizes the investor's utility. In other words, the portfolio that can maximize the Sharpe ratio is the optimal portfolio for investors. The investor will only select portfolios on the efficient frontier, where the expected return is higher than the MVP. The portfolio on the efficient frontier that the investor will invest in also depends on how much risk he or she is willing to bear. Also, the optimal portfolio will be on the efficient frontier for any mean-variance investors. In this case, in terms of minimizing risks, investing TD for 19.74%, PG for 28.52%, JNJ for 20.77%, and CL for 21% is the optimal portfolio, which is under the fourth constraint. It has a return of 10.31%, a standard deviation of 11.25%, and a sharp ratio of 0.92. To maximize return, the weights of the optimal portfolio for Nividia Corporation, Cisco Systems, Intel Corporation, GS, U.S. Bancorp, TD Bank, The Allstate Corporation, P&G, J&J, and CL are 18.29%, 0.18%, -6.66%, 9.54%, 10.90%, 42.02%, 4.95%, 48.64%, 38.88%, and 18.25% respectively, with a return of 16.87%, standard deviation of 15.04%, and Sharpe ratio of 1.12.

Table 1. The results of the Full Markowitz Model

MM (Constr1):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	39.46%	-3.04%	3.00%	1.32%	6.08%	-0.44%	19.33%	11.31%	25.51%	18.85%	19.40%	8.91%	10.93%	81.54%
MaxSharpe	43.46%	14.28%	1.09%	5.44%	3.66%	6.72%	35.15%	0.79%	43.96%	30.98%	14.45%	15.06%	13.59%	110.85%
MM (Constr2):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	39.46%	-3.04%	3.00%	1.32%	6.08%	-0.44%	19.33%	11.31%	25.51%	18.85%	19.40%	8.91%	10.93%	81.54%
MaxSharpe	85.00%	18.29%	0.18%	6.66%	9.54%	10.90%	42.02%	4.95%	48.64%	38.88%	18.25%	16.87%	15.04%	112.17%
MM (Constr3):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	39.46%	-3.04%	3.00%	1.32%	6.08%	-0.44%	19.33%	11.31%	25.51%	18.85%	19.40%	8.91%	10.93%	81.54%
MaxSharpe	85.00%	18.29%	0.18%	6.66%	9.54%	10.90%	42.02%	4.95%	48.64%	38.88%	18.25%	16.87%	15.04%	112.17%
MM (Constr4):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	9.98%	0.00%	0.00%	0.00%	0.00%	0.00%	19.74%	0.00%	28.52%	20.77%	21.00%	10.31%	11.25%	91.63%
MaxSharpe	0.00%	9.32%	0.00%	0.00%	0.00%	0.00%	23.96%	0.00%	40.43%	17.53%	8.76%	13.06%	12.66%	103.17%
MM (Constr5):	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
MinVar	0.00%	-0.98%	0.07%	2.54%	1.04%	3.47%	24.76%	-7.90%	28.58%	25.78%	24.73%	10.15%	11.17%	90.85%
MaxSharpe	0.00%	12.64%	5.88%	8.30%	1.30%	2.55%	29.86%	-2.85%	40.91%	24.10%	8.27%	13.88%	13.06%	106.24%

5. Conclusion

This paper is aimed to find out the most efficient portfolio for the investor using the Full Markowitz Model. The MM model helps the investor to find out the efficient set of the portfolio by finding out the trade-off between risk and return. In general, the focus of MPT is to find the optimal portfolio all agents would want to hold.

By using the MM model, it can be found that the portfolio under the fourth constraint is the optimal choice when the investors want to minimize risks, that is minimizing variance in the Full Markowitz Model. The optimal portfolio, in this case, includes TD for 19.74%, PG for 28.52%, JNJ for 20.77%, and CL for 21%. It has a return of 10.31%, a standard deviation of 11.25%, and a sharp ratio of 0.92. For investors who prefer to maximize return, investing Nividia Corporation, Cisco Systems, Intel

Corporation, GS, U.S. Bancorp, TD Bank, The Allstate Corporation, P&G, and J&J, and CL for 18.29%, 0.18%, -6.66%, 9.54%, 10.90%, 42.02%, 4.95%, 48.64%, 38.88%, and 18.25% respectively is the optimal choice, which has a return of 16.87%, standard deviation of 15.04%, and Sharpe ratio of 1.12.

However, limitations exist if everyone demands the same assets to build their portfolio because it will impact the equilibrium asset prices of these individual assets. CAPM model developed by Sharpe can help to deal with the problems and show the relative risk of security as compared to the market through beta. Also, Index Model can be used in future studies to compare the results to give better suggestions for investors' decision-making.

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